MATH A290H: INTRODUCTION TO TENSORS AND CALCULUS ON MANIFOLDS HONORS

Item	Value
Curriculum Committee Approval Date	12/02/2020
Top Code	170100 - Mathematics, General
Units	5 Total Units
Hours	90 Total Hours (Lecture Hours 90)
Total Outside of Class Hours	0
Course Credit Status	Credit: Degree Applicable (D)
Material Fee	No
Basic Skills	Not Basic Skills (N)
Repeatable	No
Grading Policy	Standard Letter (S), • Pass/No Pass (B)
Associate Arts Local General Education (GE)	• OC Comm/Analytical Thinking - AA (OA2)
Associate Science Local General Education (GE)	• OCC Comm/AnalyticalThinking-AS (OAS2) • OCC Mathematics (OMTH)
California General Education Transfer Curriculum (Cal-GETC)	• Cal-GETC 2A Math Concepts (2A)
Intersegmental General Education Transfer Curriculum (IGETC)	• IGETC 2A Math Concepts (2A)
California State University General Education Breadth (CSU GE-Breadth)	• CSU B4 Math/Quant.Reasoning (B4)

Course Description

Introductory study of elementary tensor algebra and calculus, differential and integral calculus in higher dimensions, differential forms, and calculus on manifolds. PREREQUISITE: MATH A280 or MATH A280H; and MATH A285 or MATH A285H. Transfer Credit: CSU; UC.

Course Level Student Learning Outcome(s)

1. Use elementary tensor methods for calculating generalized derivatives in Riemannian spaces.
2. Demonstrate the ability of proving theorems for the calculus of Rn.
3. Employ the exterior calculus in generalized Stokes' theorems, especially as applicable to manifold theory.

Course Objectives

- 1. Use the Einstein summation convention.
- 2. Use basic linear algebra for tensors.
- 3. Employ elementary tensor coordinate transformations.
- 4. Perform tests for tensor character.
- 5. Discuss the metric tensor and its relation to inner products.
- 6. Use the calculus of tensor derivatives.
- 7. Explain the relationship between tensors and Reimannian curves and Riemannian curvature.
- 8. Discuss proofs and uses of the inverse and implicit function theorems for Rn.
- 9. Employ elementary sets of measure zero.
- 10. Prove the change of variable theorem.
- 11. Use fields and differential forms.
- 12. Explain integration on chains and its relationship to the fundamental theorem of calculus.
- 13. Employ simple manifolds and elementary tensor fields on manifolds.
- 14. Prove and use the generalized Stokes theorem on manifolds.
- 15. Explain the relationship between the generalized Stokes theorem and classical theorems.
- 16. Prove simple theorems about the topology of Rn including notions of interior, exterior, boundary, compactness and connectedness.
- 17. Prove the Fubini theorem for Rn.
- 18. Employ elementary partitions of unity.

Lecture Content

It is imperative that instructors cover all topics in the outline. The instructor may determine the order of topics. 1. Review Exterior Algebra a. duel spaces and the calculus b. bases c. formalism d. adjoints e. tangent bundles f. calculus on tangent bundles 2. Tensor Calculus a. The Einstein notation b. linear algebra for tensors c. contravariant and covariant coordinates d. types of tensors e. invariants f. operations g. the metric tensor h. Christoffel symbols i. covariant differentiation j. absolute differentiation k. tensors and dual spaces l. introduction to manifolds m. tensors on manifolds 3. Euclidean Space a. norm and inner product b. compactness and the finite Tychonoff theorem c. continuity and oscillation 4. Differentiation a. definition and uniqueness b. basic theorems and proofs c. differentiability criterion d. inverse function theorem e. implicit function theorem 5. Integration a. definition and fundamental criterion b. measure zero c. integrability d. Fubinis theorem e. partitions of unity f. change of variable theorem 6. Integration on Chains a. more exterior algebra b. tangent spaces and differential forms c. singular n cubes and boundaries d. functional theorem (generalized Greens) 7. Review of Differential Geometry a. curves and Frenet formulas b. definition of surfaces c. fundamental forms d. Gaussian curvature e. introduction to Riemannian geometry 8. Integration on Manifolds a. definitions and their equivalences b. fields and forms on manifolds c. generalized S tokes theorem d. elements of hypervolume e. classical theorems

Method(s) of Instruction

- Lecture (02)

Instructional Techniques

Lecture, discussion

Reading Assignments

As assinged from textbooks. 1 hour

Writing Assignments

Students write definitions, theorems, proofs, and justifications. 1 hour

Out-of-class Assignments

Students write definitions, theorems, proofs, and justifications. 8 hour

Demonstration of Critical Thinking

Problem solving exercises, theorems, proofs, justifications

Required Writing, Problem Solving, Skills Demonstration

Students write definitions, theorems, proofs, and justifications

Eligible Disciplines

Mathematics: Masters degree in mathematics or applied mathematics OR bachelors degree in either of the above AND masters degree in statistics, physics, or mathematics education OR the equivalent. Masters degree required.

Textbooks Resources

1. Required Kay, David. Schaums Outline of Tensor Analysis, ed. New York: McGraw Hill, 2011 Rationale: . 2. Required Spivak, Michael. Calculus on Manifolds, ed. Atlanta: Perseus Publishing, 1973 Rationale: .

Other Resources

1. Other appropriate textbook as chosen by faculty.